CHAPTER 7 DYNAMIC MODELLING AND CONTROL OF DC/DC CONVERTERS

7.1. Introduction

Characteristics of DC/DC converters described in CHAPTER 2 to CHAPTER 5 are under steady state condition with open-loop control. The converters has run for a considerable time to settle down to a stable condition with their regular gate signals. Actually, the steady state analysis is for designing the converters. Also, practical converters seldom work in open-loop. Closed-loop control is usually applied in power electronic products so that their outputs (output voltage, output current or input current) maintain regulated to desired levels even with the change of conditions such as variation of load, input voltage and voltage drop from power loss, etc. Fig. 7.1 shows a basic block diagram of a closed-loop controlled system.

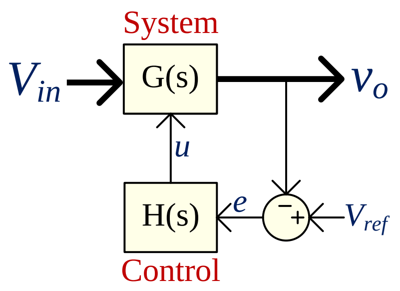


Fig. 7.1. Basic block diagram of a closed-loop control system

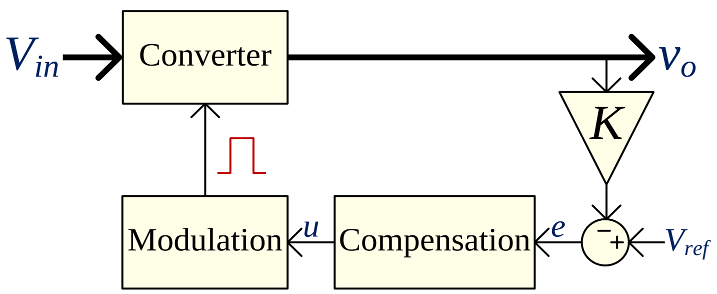


Fig. 7.2. Block diagram of a signal loop voltage controlled DC/DC converter

Designing closed-loop control system, dynamic models, or called small-signal models, of the converters have to be obtained in the first place which are mathematical models, in the form of transfer functions, of the converter with the response characteristics from applying small-signal. Fig. 7.2 shows a block diagram of a signal loop voltage controlled DC/DC converter. DC/DC converters are highly non-linear and have the features of both analogue and digital systems. They are controlled by digital type of duty ratio of gate signals for most converters and by phase difference of gate signals for phase shift converters. As a results, state-space average technique is usually used for dynamic modelling. Different characteristics of the small-signal models require different control method for good stability of the whole power electronic systems.

7.2. General State-space Averaging Technique

Dynamic models of power converters can be obtained by using state-space average technique. The following explanation and description of state-space average technique are based on deriving generalized equations for basic power converters operating in continuous mode. Without considering charging and discharging of output filtering capacitors, converters in continuous mode has only two states of operation (on-state and off-state). The analysis starts from state-space equations during on-state and off-state of the transistor of a power converter. Averaging method is used for linearize them.

In on-state, the transistor of the converter conducts for a ration of D of a switching period. The state-space equation in on-state is:

$$\dot{X} = A_{on}X + B_{on}Y \tag{7.1}$$

In off-state, the main diode of the converter (such as the free-wheel diode of a buck converter) conduct for a duty ratio of (1-D). The state-space equation in off-state is:

$$\dot{X} = A_{off}X + B_{off}Y \tag{7.2}$$

where X is the state-space variable such as a set of capacitor voltage and inductor current in the form of matrix. A_{on} and B_{on}, and A_{off} and B_{off} are the state-space matrices of the converter during on-state and off-state, respectively. Y is the input variable such as input voltage, V_{in}.

Since on-state and off-state of a converter are presented for DT_S and (1-D)T_S duration, the state-space equations (7.1) and (7.2) can be averaged by the conduction ratio as:

$$\dot{X} = \left[DA_{on} + (1-D)A_{off}\right]X + \left[DB_{on} + (1-D)B_{off}\right]Y \tag{7.3}$$

Considering there is a small signal variation d to the duty ratio D, this causes a small variation x of the state variable, X. The variation of the small signal and the state-space variable can be written as:

$$D = \overline{D} + d \tag{7.4}$$

$$X = \overline{X} + x \tag{7.5}$$

where $\overline{X}$ is the DC or steady-state component, x is a small-signal variation of $\overline{X}$, and d is a small signal variation in the steady-state or DC component of duty ratio of the gate signal of $\overline{D}$. From equations (7.4) and (7.5), equations (7.3) can be rewritten as:

$$\dot{\overline{X}} + \dot{x} = \left[(\overline{D}+d)A_{on} + (1-\overline{D}-d)A_{off}\right](\overline{X}+x) + \left[(\overline{D}+d)B_{on} + (1-\overline{D}-d)B_{off}\right]Y \tag{7.6}$$

Expanding equations (7.6), it gives:

$$\begin{aligned}\dot{\overline{X}} + \dot{x} = {} & \left[\overline{D}A_{on} + (1-\overline{D})A_{off}\right]\overline{X} + (dA_{on} - dA_{off})\overline{X} \\ & + \left[\overline{D}A_{on} + (1-\overline{D})A_{off}\right]x + (dA_{on} - dA_{off})x \\ & + \left[\overline{D}B_{on} + (1-\overline{D})B_{off}\right]Y + (dB_{on} - dB_{off})Y\end{aligned} \tag{7.7}$$

If the small-signal is zero, then:

$$\dot{\overline{X}} = \left[\overline{D}A_{on} + (1-\overline{D})A_{off}\right]\overline{X} + \left[\overline{D}B_{on} + (1-\overline{D})B_{off}\right]Y \tag{7.8}$$

Eliminating equation (7.8) from (7.7):

$$\dot{x} = (dA_{on} - dA_{off})\overline{X} + \left[\overline{D}A_{on} + (1-\overline{D})A_{off}\right]x + (dA_{on} - dA_{off})x + (dB_{on} - dB_{off})Y \tag{7.9}$$

Neglecting the high order small signal variation, then:

$$\dot{x} = (dA_{on} - dA_{off})\overline{X} + (dB_{on} - dB_{off})Y + \left[\overline{D}A_{on} + (1-\overline{D})A_{off}\right]x \tag{7.10}$$

$$\dot{x} = Ax + Fd \tag{7.11}$$

where

$$A = \overline{D}A_{on} + (1-\overline{D})A_{off} \tag{7.12}$$

$$F = (A_{on} - A_{off})\overline{X} + (B_{on} - B_{off})Y \tag{7.13}$$

A is the average of A_{on} and A_{off}. F is the difference of equations (7.1) and (7.2) which is the difference between the on-state and the off-state of X. Equation (7.11) is now a linearized equation and the original digital feature is removed. The equation can be solved by Laplace Transform to give a transfer function:

$$\frac{x}{d} = [sI - A]^{-1}F \tag{7.14}$$

where I is a unit matrix. $[sI - A]^{-1}$ is the inverse of $[sI - A]$.

7.3. Small-signal modelling of DC/DC Converters in Continuous Mode

Small-signal models of three basic DC/DC converters obtained by state-space average technique is explained in the followings. These dynamic models help to learn the dynamic response of the converters and to design their corresponding closed-loop controllers.

7.3.1. Small-signal Modelling of Buck Converters in Continuous Mode

7.3.1.1. State-space Equations of Buck Converters in Continuous Mode

(a) On-state (b) Off-state

Fig. 7.3. Equivalent circuits of a buck converter in continuous mode

Equivalent circuits of a buck converter in on-state and off-state are shown in Fig. 7.3. T is the transistor and D is the free-wheel diode of the converter. State-space variable of the converter is taken as the set of the inductor current, i_L, and the output filtering capacitor voltage, v_C which is also the output voltage. i_o is the output current and is equal to v_C/R. d is the small-signal variation of the duty ratio of the gate signal.

When T is on:

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in} \equiv \dot{X} = A_{on} X + B_{on} Y \tag{7.15}$$

When T is off:

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_{in} \equiv \dot{X} = A_{off} X + B_{off} Y \tag{7.16}$$

Linearizing the above state-space equations of the buck converter with Equation (7.8), then:

$$\begin{bmatrix} \dot{\hat{i}}_L \\ \dot{\hat{v}}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{v}_C \end{bmatrix} + \begin{bmatrix} \frac{V_{in}}{L} \\ 0 \end{bmatrix} d \equiv \dot{x} = Ax + Fd \tag{7.17}$$

where $\hat{i}_L$, $\hat{v}_C$, and d are small-signal perturbations about the average DC or steady-state values of i_L, v_C, and D, respectively.

Solving equation (7.17) by Laplace Transform, it gives:

$$\frac{x}{d}=\frac{\begin{bmatrix}\dot{\hat{i}}_L\\ \hat{v}_C\end{bmatrix}}{d}=\frac{adj[sI-A]F}{\det[sI-A]}=\frac{\begin{bmatrix}s+\frac{1}{RC} & -\frac{1}{L}\\ \frac{1}{C} & s\end{bmatrix}\begin{bmatrix}\frac{1}{L}\\ 0\end{bmatrix}V_{in}}{s^2+\frac{s}{RC}+\frac{1}{LC}} \tag{7.18}$$

Therefore the control-to-output small-signal transfer functions of a buck converter in continuous mode are:

$$\frac{\hat{v}_C}{d}=\left(\frac{V_{in}}{LC}\right)\left(\frac{1}{s^2+\frac{s}{RC}+\frac{1}{LC}}\right) \tag{7.19}$$

$$\frac{\hat{i}_L}{d}=\left(\frac{V_{in}}{L}\right)\left(\frac{s+\frac{1}{RC}}{s^2+\frac{s}{RC}+\frac{1}{LC}}\right) \tag{7.20}$$

7.3.1.2. Characteristics of Small Signal Response of Buck Converters in Continuous Mode

Equations (7.19) and (7.20) are both second order transfer functions of buck converters in frequency domain. They have two left-half plane conjugate poles. Equation (7.19) is the output voltage transfer function without zero whereas the current transfer function, Equation (7.20) has one left-half-plane zero. A frequency response bode plot of the voltage transfer function of a buck converter in continuous mode is shown in Fig. 7.4. Bold plot can be generated from substituting $j\omega$ to s in the transfer functions. The parameters of this converter are that L is 1mH, C is 10μF, R is 10Ω and V_{in} is 10V.

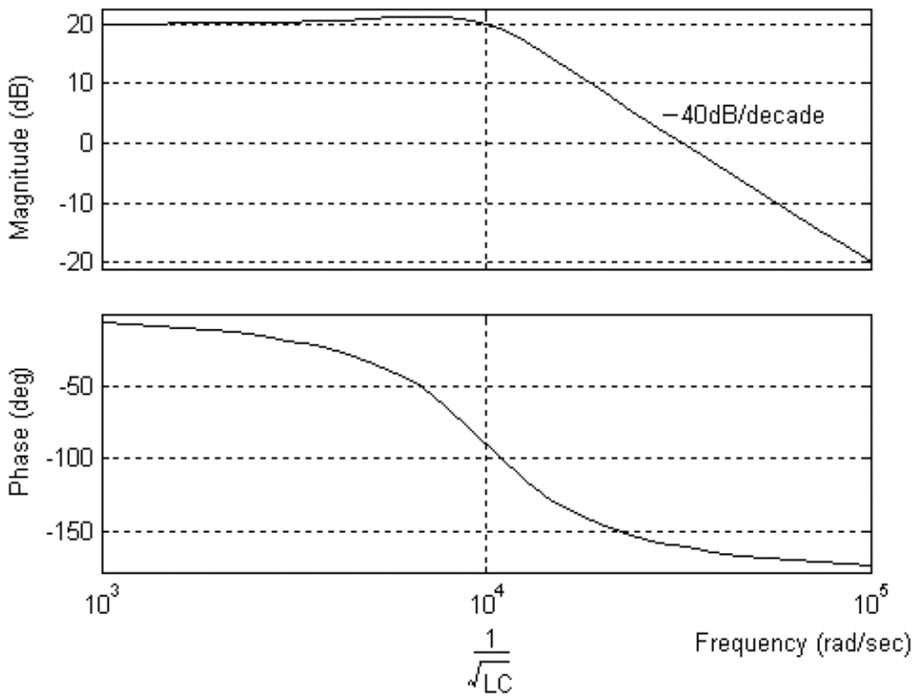


Fig. 7.4. Bode plot of a small signal transfer function (duty ratio to output voltage) of a buck converter in continuous mode

The plot shows after the corner frequency, the gain of the transfer function is damped in a -40dB/decade manner. The total phase shift of the transfer function is 180°. Ideally, closed-loop control of the output voltage is simple with buck converters as a proportional feedback can ensure stability. In practice, the frequency response will be more heavily damped because of the circuit losses. The poles will move further into the left-plane. If under light load, the loop phase shift approaches 180°at high frequency, therefore a lead-lag compensation or an integrator with proportion (Integral-proportional Control, PI Control) will be commonly used to improve the phase shift near the crossover frequency (the frequency shown in the bode plot which the magnitude is 0dB).

7.3.2. Small-signal Modelling of Boost Converters in Continuous Mode

7.3.2.1. State-space Equations of Boost Converters in Continuous Mode

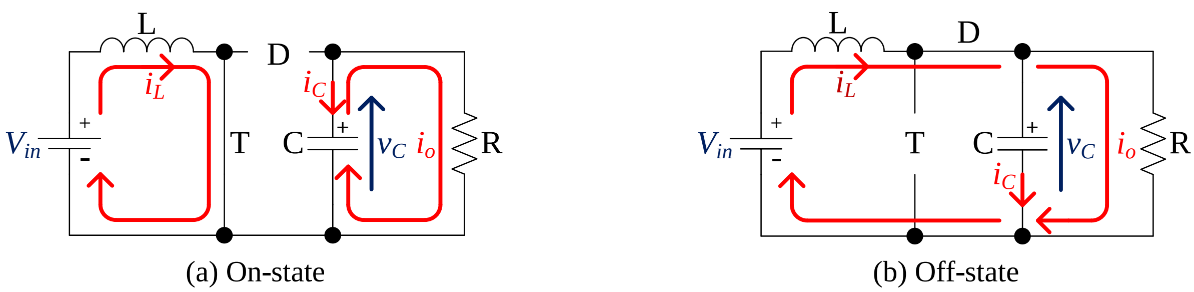


Fig. 7.5. Equivalent circuits of a boost converter in Continuous Mode

Boost converters are popular especially for power factor correction. Parameters for the energy storage elements, i_L and v_C, are for the state-space variable. Method for obtaining small-signal transfer functions of boost converters in continuous mode is similar to that of buck converters. Fig. 7.5 shows the equivalent circuits of a boost converter in continuous mode.

When T is on:

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in} \equiv \dot{X} = A_{on} X + B_{on} Y \tag{7.21}$$

When T is off:

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in} \equiv \dot{X} = A_{off} X + B_{off} Y \tag{7.22}$$

Linearizing the above state-space equations of the buck converter with Equation (7.8), then:

$$\begin{bmatrix} \dot{\hat{i}}_L \\ \dot{\hat{v}}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1-D}{L} \\ \frac{1-D}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{v}_C \end{bmatrix} + \begin{bmatrix} \frac{v_C}{L} \\ -\frac{i_L}{C} \end{bmatrix} d \equiv \dot{x} = Ax + Fd \tag{7.23}$$

where $\hat{i}_L$, $\hat{v}_C$, and d are small-signal perturbations about the average DC or steady-state values of i_L, v_C, and D, respectively.

Solving equation (7.23) by Laplace Transform, it gives:

$$\frac{x}{d}=\frac{\begin{bmatrix}\dot{\hat{i}}_L\\ \hat{v}_C\end{bmatrix}}{d}=\frac{adj[sI-A]F}{\det[sI-A]}=\frac{\begin{bmatrix}s+\frac{1}{RC} & -\frac{1-D}{L}\\ \frac{1-D}{C} & s\end{bmatrix}\begin{bmatrix}\frac{v_C}{L}\\ -\frac{i_L}{C}\end{bmatrix}}{s^2+\frac{s}{RC}+\frac{(1-D)^2}{LC}} \tag{7.24}$$

The output voltage and input current frequency responses of a boost converter in continuous mode are:

$$\frac{\hat{v}_C}{d}=\frac{\frac{1-D}{LC}v_C-\frac{i_L}{C}s}{s^2+\frac{s}{RC}+\frac{(1-D)^2}{LC}} \tag{7.25}$$

$$\frac{\hat{i}_L}{d}=\frac{\frac{v_C}{L}s+\frac{1}{LC}\left[\frac{v_C}{R}+i_L(1-D)\right]}{s^2+\frac{s}{RC}+\frac{(1-D)^2}{LC}} \tag{7.26}$$

Equations (7.25) and (7.26) can be simplified by replacing i_L and v_C using the relationship:

$$i_L=\frac{I_o}{1-D}=\frac{v_C}{(1-D)R}\ \text{and}\ v_C=\frac{V_{in}}{1-D} \tag{7.27}$$

Hence, the control to output voltage transfer and the control to inductor current (input current) transfer functions of boost converter in continuous mode are:

$$\frac{\hat{v}_C}{d}=\frac{V_{in}}{RC(1-D)^2}\left[\frac{\frac{R(1-D)^2}{L}-s}{s^2+\frac{s}{RC}+\frac{(1-D)^2}{LC}}\right] \tag{7.28}$$

$$\frac{\hat{i}_L}{d}=\frac{V_{in}}{L(1-D)}\left[\frac{s+\frac{2}{RC}}{s^2+\frac{s}{RC}+\frac{(1-D)^2}{LC}}\right] \tag{7.29}$$

7.3.2.2. Characteristics of Small Signal Response of Boost Converters in Continuous Mode

The system of a boost converter in continuous mode is second order with two left-half plane conjugate poles. The output voltage transfer function has a right-half plane zero. A frequency response bode plot of a boost converter in continuous mode is shown in Fig. 7.6. The parameters of the boost converter in this plot are that L is $1mH$, C is $10\mu F$, R is 10Ω, V_{in} is $10V$ and D is 0.5. Because of the right-half plane zero, the simple voltage feedback control is very difficult. The phase shift of the transfer function is large and around 180° to 270°. Using conventional compensation control method will cause the system to be unstable. Hence, using an inner current loop in the system of the boost converter is necessary to improve the stability (margin) of the system.

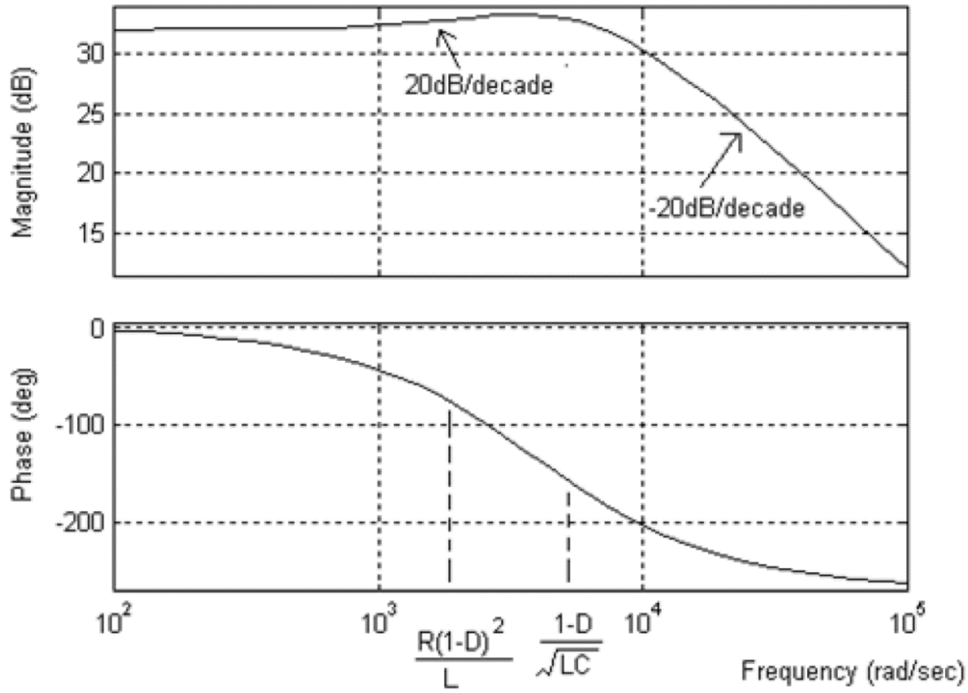


Fig. 7.6. Bode plot of a small signal transfer function (duty ratio to output voltage) of a boost converter in continuous mode

7.3.3. *Small-signal Modelling of Buck-boost Converters in Continuous Mode*

7.3.3.1. State-space Equations of Buck-Boost Converters in Continuous Mode

(a) On-state (b) Off-state

Fig. 7.7. Equivalent circuits of a buck-boost converter in continuous mode

Parameters for the energy storage elements, i_L and v_C, are for the state-space variable. Method for obtaining small-signal transfer functions of buck-boost converters in continuous mode is similar to that of boost converters. Fig. 7.7 shows the equivalent circuits of a buck-boost converter in continuous mode.

When T is on:

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in} \equiv \dot{X} = A_{on} X + B_{on} Y \tag{7.30}$$

When T is off:

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_{in} \equiv \dot{X} = A_{off} X + B_{off} Y \tag{7.31}$$

Linearizing the above state-space equations of the buck-boost converter with Equation (7.8), then:

$$\begin{bmatrix} \dot{\hat{i}}_L \\ \dot{\hat{v}}_C \end{bmatrix} = \begin{bmatrix} 0 & -\dfrac{1-D}{L} \\ \dfrac{1-D}{C} & -\dfrac{1}{RC} \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{v}_C \end{bmatrix} + \begin{bmatrix} \dfrac{v_C + V_{in}}{L} \\ -\dfrac{i_L}{C} \end{bmatrix} d \equiv \dot{x} = Ax + Fd \tag{7.32}$$

where $\hat{i}_L$, $\hat{v}_C$, and d are small-signal perturbations about the average DC or steady-state values of i_L, v_C, and D, respectively.

Solving equation (7.32) by Laplace Transform, it gives:

$$\frac{x}{d} = \frac{\begin{bmatrix} \dot{\hat{i}}_L \\ \hat{v}_C \end{bmatrix}}{d} = \frac{adj[sI - A]F}{\det[sI - A]} = \frac{\begin{bmatrix} s + \dfrac{1}{RC} & -\dfrac{1-D}{L} \\ \dfrac{1-D}{C} & s \end{bmatrix} \begin{bmatrix} \dfrac{v_C + V_{in}}{L} \\ -\dfrac{i_L}{C} \end{bmatrix}}{s^2 + \dfrac{s}{RC} + \dfrac{(1-D)^2}{LC}} \tag{7.33}$$

The output voltage and input current frequency responses of the converter in continuous mode are:

$$\frac{\hat{v}_C}{d} = \frac{\dfrac{(1-D)(v_C + V_{in})}{LC} - \dfrac{i_L}{C} s}{s^2 + \dfrac{s}{RC} + \dfrac{(1-D)^2}{LC}} \tag{7.34}$$

$$\frac{\hat{i}_L}{d} = \frac{\dfrac{v_C + V_{in}}{L} s + \dfrac{1}{LC}\left[\dfrac{v_C + V_{in}}{R} + i_L(1-D)\right]}{s^2 + \dfrac{s}{RC} + \dfrac{(1-D)^2}{LC}} \tag{7.35}$$

Equations (7.34) and (7.35) can be simplified by replacing i_L and v_C using the relationship:

$$i_L = \frac{DV_{in}}{(1-D)^2 R} \text{ and } v_C = \frac{DV_{in}}{1-D} \tag{7.36}$$

Hence, the small-signal transfer functions of a buck-boost converter in continuous mode are:

$$\frac{\hat{v}_C}{d} = \frac{DV_{in}}{RC(1-D)^2}\left[\frac{\dfrac{R(1-D)^2}{DL} - s}{s^2 + \dfrac{s}{RC} + \dfrac{(1-D)^2}{LC}}\right] \tag{7.37}$$

$$\frac{\hat{i}_L}{d} = \frac{V_{in}}{L(1-D)}\left[\frac{s + \dfrac{1+D}{RC}}{s^2 + \dfrac{s}{RC} + \dfrac{(1-D)^2}{LC}}\right] \tag{7.38}$$

7.3.3.2. Characteristics of Small Signal Response of Buck-boost Converters in Continuous Mode

The system of a buck-boost converter in continuous mode is second order with two left-half plane conjugate poles which is similar to a boost converter. A frequency response plot of a buck-boost converter in continuous mode is shown in Fig. 7.8. The parameters of the boost converter in this plot are that L is $1mH$, C is $5\mu F$, R is 30Ω, V_{in} is $10V$ and D is 0.5. Because of the right-half plane zero, the system is difficult to be controlled with simple voltage feedback control. Since the phase shift of the transfer function is large and around 180° to 270°, conventional compensation will cause the system to be unstable. As a result, an inner current loop is necessary to improve the stability of the system of a buck-boost converter. The situation of a buck-boost converter in terms of stability is similar to a boost converter.

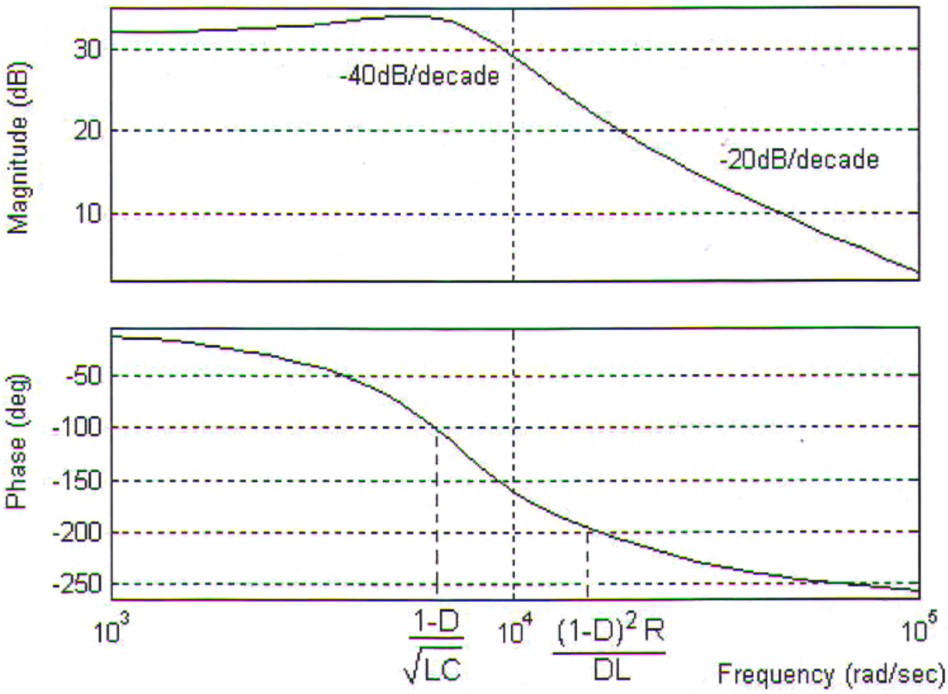


Fig. 7.8. Bode plot of a small signal transfer function (duty ratio to output voltage) of a buck-boost converter in continuous mode

7.3.4. Small-signal Modelling of Isolated DC/DC Converters

Forward converters are developed from buck converters. Also, flyback converters are developed from buck-boost converters. Applying equivalent circuits of lossless transformers and using state-space technique, small-signal transfer functions can be obtained.

7.4. Control of DC/DC Converters

The small-signal frequency response introduced in **Chapter 7.3** describes only the open-loop characteristics of the DC/DC converters. Operating the converters with certain conditions such as desired Output voltage, output current or input current, designing and building their closed-loop control system is necessary. Stability of a system can be assessed by the amount of gain margin and phase margin of the system. Accepted norm for an unconditionally stable linear system is to have a gain margin of 6dB and a phase margin of 45°. The closed-loop cross-over frequency, f_{xo}, must be less than 20% of the switching frequency of a converter for ensuring the error amplifier of a compensation network will not be amplified significantly. Also, the DC gain has to be designed to be high so that the steady-state error is minimized.

7.4.1. Voltage Mode Control of DC/DC Converters

A buck converter or a step-down converter in continuous mode has a second order control-to-output transfer function without zeros. The maximum loop phase shift is there 180°. Output voltage feedback may be easily applied to the converter providing some sort of simple lead-lag compensation, proportional-integral (PI) feedback or even a proportional feedback can be included in the loop. A typical single voltage loop for buck converter is shown in Fig. 7.9. The closed-loop control including a potential divider for stepping down the voltage from high output voltage of the converter, an error amplifier for comparing the output with the reference and for compensation, a voltage comparator for pulse-width modulation to produce gate signal by comparing the control signal from the error amplifier with a sawtooth voltage waveform. The switching frequency of the converter is equal to the frequency of the sawtooth waveform.

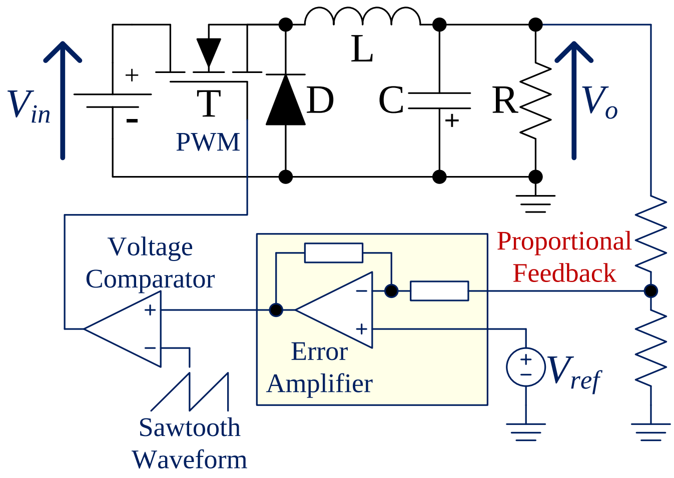


Fig. 7.9. A typical voltage mode control system of a buck converter

7.4.2. Current Mode Control of DC/DC Converters

As both boost and buck-boost converters have a right-half plane zero, the phase shift of the systems is large. The maximum phase shift of these converter system is 270° which is very difficult to use typical compensation techniques for the feedback loop. Hence, boost and buck-boost converters usually have an inner current loop for the closed-loop systems. The inductor current of a converter is fed to the control system. Its average current is controlled in average current mode control and its peak current is controlled in peak current mode control. Inner current loops in closed-loop systems are not only for DC/DC converters but also very commonly used in Power Factor Correction Converters (PFCC).

7.4.2.1. Average Current Mode Control

Fig. 7.10 shows a boost converter with an inner current loop for average current control. Inductor current of the converter is sensed by a current transducer, such as a hall effect current transducer, or a current sensing resistor which is a non-inductive low resistance shunt resistor so that the current signal is converted to be in a voltage form and is fed to the controller. In Fig. 7.10, the current signal is fed to an error amplifier. As a result, the inductor current, i_L, is controlled and the average inductor current is equal to the reference current, I_{ref}, in steady state.

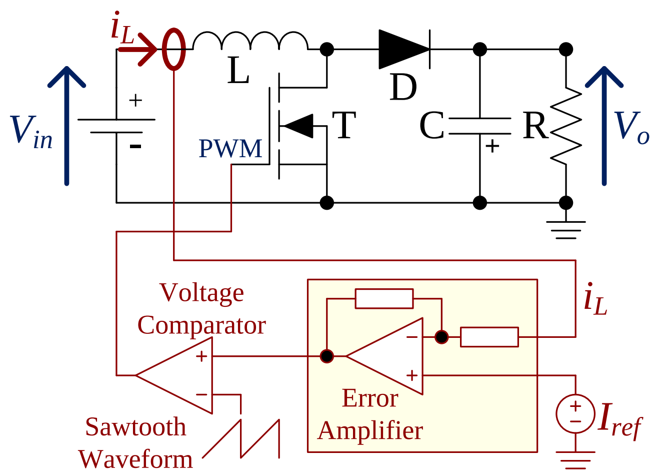


Fig. 7.10. An inner average current loop of a boost converter

7.4.2.2. Peak Current Mode Control

Fig. 7.11 shows a peak current mode control system in a flyback converter. The inner current loop of this system consists of a flip-flop, a voltage comparator, a current sensing resistor and a fixed frequency clock signal. In an entire closed-loop control system for controlling output voltage of a converter, the inner current loop is connected to the voltage loop where the voltage loop produces a reference current, I_{ref}, to the current loop.

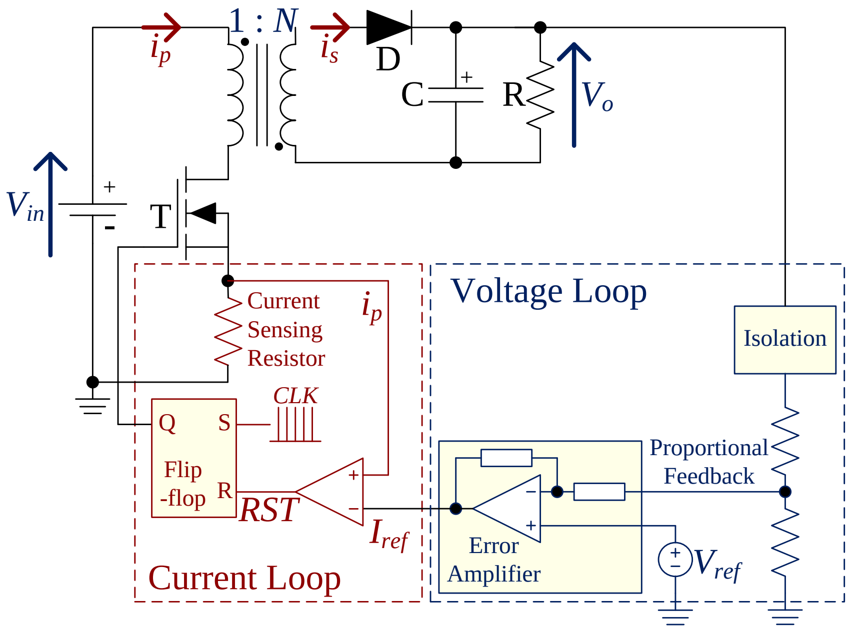


Fig. 7.11. A flyback converter with peak current mode control

In the beginning of a switching cycle, the output signal of the clock, *CLK*, sets the output of the flip-flop (or latch), Q, to high level. The high level latch output is for switching on the transistor of the converter. Once the transistor is switched on, the inductor current, i_L, increases linearly and the diode, D, becomes reverse biased.

In this example, the current of the primary winding of the high frequency transformer, i_p, is transduced to be in the format of voltage by sensing the voltage drop of a current sensing resistor (usually called shunt resistor). Avoiding high power loss and performance of the converter, the resistance is of the current sensing resistance is non-inductive and with low resistance, typically 0.5Ω or less.

The current feedback signal is compared with I_{ref} by a voltage comparator. i_L increases linearly after switched on the transistor by setting the flip-flop. When i_L is just higher than I_{ref}, the output of the voltage comparator, *RST*, becomes in high level. This high level signal resets the flip-flop so that Q becomes in low level and the transistor is switched off. The transistor is switched on again when flip-flop is set by the clock in next switching cycle. Fig. 7.12 shows the current loop waveforms.

For isolated converter, the output voltage feedback signal has to be electrical isolated between the voltage output of the converter and the controller. Optocouplers are usually used for electrical isolation of signals.

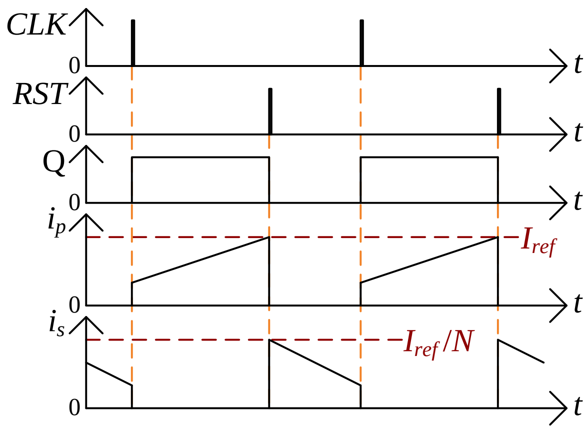


Fig. 7.12. Idealised waveforms of a current loop of a flyback converter with peak current mode control

7.4.2.3. Frequency Response of Current Mode Control

A boost converter is used as an example for explaining the frequency response of current mode control of a DC/DC converter. Output voltage transfer function of a boost converter is using *d* to control the output voltage. Implementing current mode control, I_{ref} has become the control signal. From Equation (7.23),

$$\dot{\hat{i}}_L = -\frac{1-D}{L}\hat{v}_C + \frac{v_C}{L}d \tag{7.39}$$

$$d = \frac{L}{v_C}\left(\dot{\hat{i}}_L + \frac{1-D}{L}\hat{v}_C\right) \tag{7.40}$$

When the converter is operating with current mode control, the inductor current, i_L, is equal to I_{ref}. Then:

$$d = \frac{L}{v_C}\left(\dot{\hat{i}}_{ref} + \frac{1-D}{L}\hat{v}_C\right) \tag{7.41}$$

Eliminating *d* of Equation (7.23), the equation is rewritten as:

$$\begin{bmatrix}\dot{\hat{i}}_L \\ \hat{v}_C\end{bmatrix} = \begin{bmatrix} 0 & -\frac{1-D}{L} \\ \frac{1-D}{C} & -\frac{1}{RC}\end{bmatrix}\begin{bmatrix}\hat{i}_L \\ \hat{v}_C\end{bmatrix} + \begin{bmatrix}\frac{v_C}{L} \\ -\frac{i_L}{C}\end{bmatrix}\left[\frac{L}{v_C}\left(\dot{\hat{i}}_{ref} + \frac{1-D}{L}\hat{v}_C\right)\right] \tag{7.42}$$

Since,

$$i_L = \frac{I_o}{1-D} = \frac{v_C}{(1-D)R} \tag{7.43}$$

Hence,

$$\begin{bmatrix}\dot{\hat{i}}_L \\ \hat{v}_C\end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{1-D}{C} & -\frac{2}{RC}\end{bmatrix}\begin{bmatrix}\hat{i}_L \\ \hat{v}_C\end{bmatrix} + \begin{bmatrix} 1 \\ -\frac{L}{(1-D)RC}\end{bmatrix}\dot{\hat{i}}_{ref} \tag{7.44}$$

Solving Equation (7.44) by Laplace Transform, it gives:

$$\begin{bmatrix}\hat{i}_L \\ \hat{v}_C\end{bmatrix} = \begin{bmatrix} s & 0 \\ \frac{1-D}{C} & s+\frac{2}{RC}\end{bmatrix}^{-1}\begin{bmatrix} 1 \\ -\frac{L}{(1-D)RC}\end{bmatrix}s\hat{i}_{ref} \tag{7.45}$$

The control to output voltage transfer function of a boost converter with current mode control is therefore:

$$\frac{\hat{v}_C}{\hat{i}_{ref}} = \frac{L}{(1-D)RC}\left[\frac{\frac{(1-D)^2 R}{L} - s}{s + \frac{2}{RC}}\right] \tag{7.46}$$

Equation (7.46) shows that the control to output transfer function of a boost converter has become to be first order after applying an inner current loop for current mode control. Although the right-half plane zero is present, the total phase shift at high frequency has been reduced from 270° to 180°. The design of the feedback control loop for the output voltage is much easier than without current loop. A simple lead-lag compensation or PI is sufficient. A bode plot of a boost converter with current loop is shown in Fig. 7.13. The parameters of the boost converter in this plot are that L is $1mH$, C is $10\mu F$, R is 10Ω, V_{in} is $10V$ and D is 0.5. The gain is increasing with frequency and reaches as stead-state gain at high frequency. In this case, the compensation for the voltage loop has to provide sufficient high frequency attenuation.

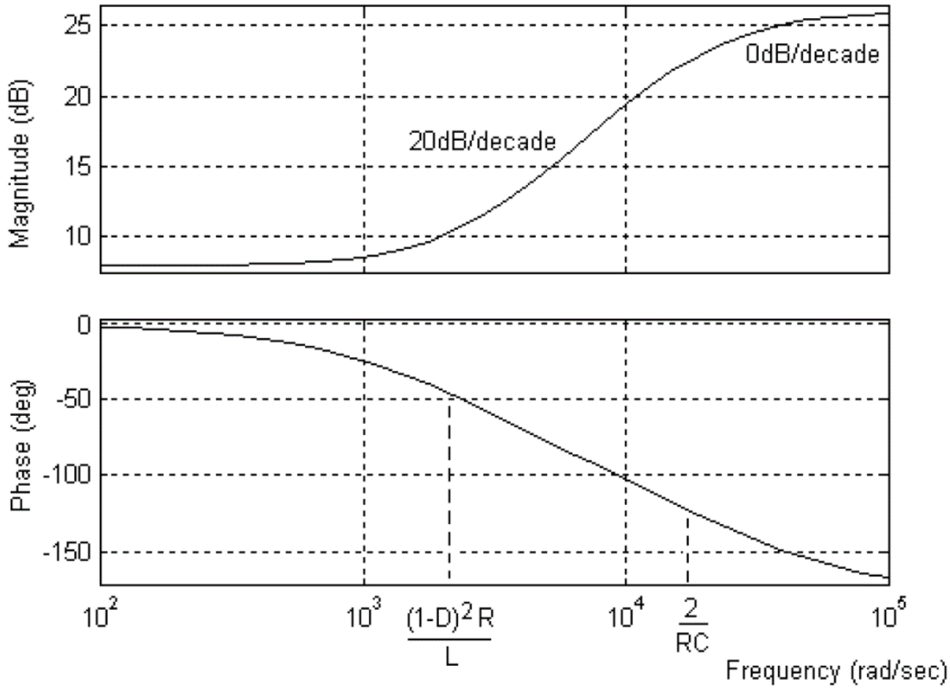


Fig. 7.13. Bode plot of a boost converter with inner current loop

7.5. Practical Control Systems of DC/DC Converterssda

7.5.1. Soft-start

Theoretically, start-up control signal of a closed-loop power converter is very high to force the output of the converter to be in steady state quickly. This may cause very high input current damage the system. Soft-start function is usually applied in a practical closed-loop control system. With soft-start function, reference voltage increases from zero to the V_{ref} gradually so that the control signal from the controller and the duty ratio of the gate signal increases gradually as well and hence, the start-up input current of the converter is limited.

7.5.2. Limit of Duty Ratio of Gate Signals

Control signal from an error amplifier or a controller may be very high, especially in start-up condition, for providing fast response of the power converter. Except buck converters, all duty ratio of gate signals for the power converters introduced in the previous chapters cannot must be less than 1, otherwise, short circuit or saturation of magnetic devices (transformers and inductors) will occur.

In a voltage mode control system, the duty ratio of the gate signal is 1 if the control signal from the error amplifier is higher than the triangular wave. For example, if the duty ratio of the gate signal is 1 in a boost converter at start-up, the output voltage will be equal to zero. The controller will receive zero output so that the controller will keep the control signal in very high level to force the response faster. The duty ratio of the gate signal will keep equal to 1. This produces very high input current so that the system will be damaged.

In a current mode control system, if the control signal is very high at start-up, the inductor current may not reach the reference current level for many switching period. Even if inductor current can increase to the reference current level, the duty ratio of the gate signal may be too high. For example, if it is a forward converter, the transformer will be saturated if the duty ratio of the gate signal is too high.

Limit of duty ratio of gate signals function in controlled power converters is necessary. Many power converter controller integrated chips (ICs) limit the maximum duty ratio of the gate signal as 0.5.

7.5.3. Electrical Isolation of Signals

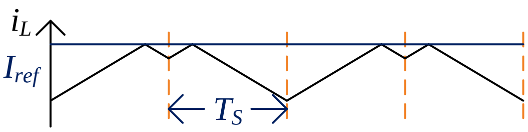
In a feedback loop of a closed-loop control system, the output is sensed and fed to the controller. For isolated power converters, electrical isolation must be provided on the output signal if the controller is on the primary side, or on the gate signal if the controller is on the secondary side. Electrical isolation of output signals can be achieved by applying optocouplers or called opto-isolators. Pulse transformer can be applied to achieve electrical isolation on gate signal as well.

7.5.4. Under-voltage Lockout (UVLO)

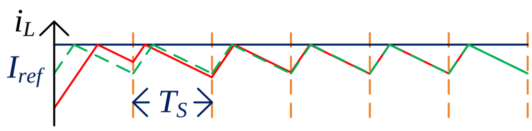
Many commercial and industrial power conversion products allow a range of input voltage, such as 100*VAC* to 240*VAC* for worldwide use. Since the input power is equal to the output power plus power losses, the input current increases if the input voltage decreases with the same output power level. Over input current may be caused if the input voltage is too low. Protecting the power conversion system, under-voltage lockout (ULVO) function is very popular in control systems and controller ICs of power converters. The input voltage is sensed. The control system will shut down the converter immediately if the input voltage is too low.

7.5.5. Slope Compensation

Peak current mode control is popular on DC/DC converters for better stability. If duty ratio of the gate signal (D) of a converter with peak current mode control in steady state is equal to or over 0.5, sub-harmonic oscillation on inductor current will occur. The fundamental frequency of the sub-harmonic oscillation of the inductor current is lower the switching frequency of the converter (called Bifurcation in this phenomena). In some cases, the frequency of the sub-harmonic oscillation is unstable as well (called Chaos in this phenomena). For example, if the output voltage of a converter is more than double of its input voltage, D is more than 0.5. The sub-harmonic oscillation causes lower and even unstable frequency on the input current and output voltage so that filtering the ripples have become more difficult. Fig. 7.14 shows the inductor current waveforms with the duty ratios equal to 0.5 and 0.25, respectively.



(a) D = 0.5



(b) D = 0.25

Fig. 7.14. Inductor current waveforms under peak current mode control

Adding a ramp to the inductor current signal can solve the problem. This is called Slope Compensation. Applying this method, the sub-harmonic oscillation problem can be solved. The more exceeding 0.5 of D, the slope of the ramp for slope compensation is higher.

7.6. Summary of Closed-loop Control Methods of DC/DC Converters

Table 7.1 shows the suggestions of control methods for DC/DC converters in continuous mode and discontinuous mode. Design of the control system is easier with the suggested control method in difficult condition of the converters. It is noted that Phase shift converters are controlled by the phase difference of their gate signals instead of PWM gate signals.

Table 7.1. Suggested control methods for DC/DC converters

Types of Converter	Modes of Operation	
	Continuous Mode	**Discontinuous Mode**
Buck	*Voltage Mode Control*	*Current Mode Control*
Forward		
Phase-shift		
Boost	*Current Mode Control*	*Voltage Mode Control*
Buck-boost		
Flyback		
Ćuk		